

# Energy and Power Management in a Series Hybrid Electric Vehicle Using Selective Evolutionary Generation

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**Abstract**—This paper applies a recently developed, on-line, search-based optimization technique called Selective Evolutionary Generation Systems (SEGS) to manage battery state-of-charge and battery and generator delivered power in a Series Hybrid Electric Vehicle (SHEV). This energy and power management problem was recently tackled with a model predictive control approach that focused on improving overall powertrain operation efficiency rather than optimizing fuel consumption. However, the resultant constrained quadratic program is not easily solvable on-line in standard automotive microcontrollers, requiring explicit solutions obtained by off-line multiparametric programming solvers for implementation. In this paper, with the same motivation of maintaining SHEV operation in the high efficiency region, we apply the SEGS algorithm to SHEV control. The SEGS algorithm is not model-based but attains globally optimal behavior on-line using a probabilistic fitness distribution over a search space. It is also optimally search efficient and tunably responsive in dynamic environments despite taking only local and computationally-inexpensive decisions. Simulation results that illustrate this SEGS application are reported.

## I. INTRODUCTION

**S**ERIES Hybrid Electric Vehicles (SHEVs) belong to the class of vehicles that combine a stored-charge battery with an internal combustion engine. They are distinguished by wheels that are driven solely by an electric motor that is in turn powered by either the engine via a generator, by the battery, or by a combination of these powerplants [1]–[3]. This engine-battery configuration not only achieves the benefits of hybrid electric vehicles, viz., an increase in fuel efficiency and a reduction of pollutant emissions [4], but the configuration also decouples the engine from the wheels thereby allowing design flexibility in locating engine and generator [1]. In the SHEV configuration, power can be drawn from the generator and engine to recharge the battery. Thus, the typical SHEV energy and power management problem is to determine how to best use the generator and battery to smoothly deliver the power at the wheels that is demanded by a driver, while maintaining the battery’s state-of-charge within acceptable limits and optimizing fuel consumption.

SHEV energy and power management is a significant problem; according to [5], there is still a requirement for sophisticated coordinated control approaches to manage multiple heterogeneous power sources in automotive powertrain architectures, and “correct estimation of the state of charge

of a battery is one of the most difficult and important research needs in battery management systems for electric and hybrid-electric vehicles” [5]. Solution techniques for the SHEV energy and power management problem take a number of forms, including rule-based control strategies [6], [7], dynamic programming [4], [8], [9], Pontryagin’s maximum principle [10], [11], equivalent consumption minimization strategies [11], [12], convex optimization [13], support vector machines [14], smoothed transitions between steady-state optimal operating points [15]–[17], online control [16], [18], Model Predictive Control (MPC) [17], stochastic MPC [19], [20], stochastic optimal control [21], etc.

Reference [17] considers a variant of the typical SHEV energy and power management problem, by seeking improved overall powertrain operation efficiency instead of optimizing fuel consumption. Unfortunately, performing optimization “in real time is, in general, impossible in standard automotive microcontrollers. One reason is the insufficient memory and computational resources of standard automotive [engine control units], which may not allow the use of advanced linear algebra libraries that speed up the solution of [the MPC optimization problem]. There are also functional reasons for not solving [the program] in real time, such as the number of operations to be performed being difficult to predict, and the implementation of advanced optimization routines being hard to validate in embedded control architectures” [17]. The deployed workaround is an explicit off-line solution of the MPC problem using multi-parametric programming that is a state feedback law computed off-line.

This paper considers a model-independent, on-line approach to SHEV energy and power management. SHEV operation is formulated similar to [17] to facilitate a similar future investigation into overall powertrain efficiency improvement, an emphasis that differs from previously-published on-line approaches [16] and [18] that optimize fuel consumption. The present work is also motivated, in part, by [21], which models SHEV operation as a controlled Markov chain using the average cost criterion to show that a stochastic control policy that yields higher probabilities of SHEV states with low cost and lower probabilities of SHEV states with high cost is an optimal control policy.

As explained in Section III, the attainment of this kind of probabilistic distribution over a set of states with (possibly time-varying) fitnesses is the goal of the recently proposed Selective Evolutionary Generation Systems (SEGS) algorithm [22], [23]. In this paper, the SEGS technique achieves the desired target probability distribution over a bounded space of SHEV states by taking only local,

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computationally-inexpensive decisions. The search space is explored using the feedback of fitness evaluation outcomes that are sensed or determined external to the method. The underlying theory [22] demonstrates that the approach is not gradient-based, but instead dwells longer in states that are more fit. The SEGS scheme also allows for tuning of the classical exploration-exploitation trade off. Further, the SEGS search is accomplished in a way that can be optimally efficient as well as adaptive to dynamic changes of cost or fitness. Search efficiency here refers to the trade off of prior information about the search space for search effort savings as quickly as possible [24]. Quick responsiveness to environment dynamics exists because of model-independence, which avoids erroneous search predispositions induced by models rendered incorrect by environment changes.

Hence, one contribution of this paper is the demonstration of a simulated real-time SHEV energy and power management control policy that is computationally-inexpensive, search efficient and responsive. Another contribution is an exploration of the effect that SEGS algorithm implementation variations have on SHEV energy and power management performance.

The remainder of this paper is as follows. Section II captures SHEV behavior for use with on-line energy and power management strategies. Section III reviews choice results of the SEGS method. Section IV applies variations of the SEGS algorithm for SHEV energy and power management. Lastly, Section V presents conclusions and future work.

## II. SHEV MODELING

Similar to [17], the power at the wheels is linked to the power of the motor by

$$P_{mot}(t) = P_{wh}(t)/\eta_{wh}(t), \quad (1)$$

where  $P_{mot}(t)$  is the power provided by the SHEV electric motor [kW],  $P_{wh}(t)$  is the power at the wheels [kW], and  $\eta_{wh}(t) \in (0, 1]$  is the (possibly time-varying) efficiency of transmitting power from SHEV motor to wheels. The motor power request must be satisfied by a combination of power supplied from the generator,  $P_{gen}(t)$  [kW], and the battery,  $P_{bat}(t)$  [kW].

At each discretized time-step  $k$ , the amount of power that was previously provided by the generator  $P_{gen}(k-1)$  must be augmented by a controller-varied step change  $\Delta P_{gen}(k)$  that yields the current delivered generator power  $P_{gen}(k)$ :

$$P_{gen}(k) = P_{gen}(k-1) + \Delta P_{gen}(k). \quad (2)$$

The power change  $\Delta P_{gen}(t)$  is upper and lower bounded by  $\Delta P_{gen \max}$  and  $\Delta P_{gen \min}$ , respectively ( $\Delta P_{gen \max} \geq 0$  and  $\Delta P_{gen \min} \leq 0$ ), to prevent abrupt generator power transitions between time-steps. The provided generator power  $P_{gen}(t)$  is also upper and lower bounded by  $P_{gen \max}$  and  $P_{gen \min}$ , respectively, because of physical limitations. After computing  $P_{gen}(k)$ , the delivered battery power at time-step  $k$  is then

$$P_{bat}(k) = P_{mot}(k)/\eta_{mot}(k) - P_{gen}(k), \quad (3)$$

where  $\eta_{mot}(k) \in (0, 1]$  is the (possibly time-step-varying) efficiency of transmitting power from SHEV battery and generator to the electric motor. When  $P_{gen}(k) > P_{mot}(k)/\eta_{mot}(k)$ , the battery charges, and when  $P_{gen}(k) < P_{mot}(k)/\eta_{mot}(k)$ , the battery supplies power in addition to that delivered by the generator. As with the generator, the output battery power  $P_{bat}(t)$  is also upper and lower bounded by  $P_{bat \max}$  and  $P_{bat \min}$ , respectively, as a result of physical power delivery and charge restrictions.

It is possible to augment the SHEV with controllable friction brakes as follows. At each discretized time-step  $k$ , the amount of power previously dissipated by these brakes  $P_{brk}(k-1)$  is affected by a controller-varied step change  $\Delta P_{brk}(k)$  that provides the current braking power  $P_{brk}(k)$ :

$$P_{brk}(k) = P_{brk}(k-1) + \Delta P_{brk}(k). \quad (4)$$

The brake power change  $\Delta P_{brk}(t)$  is upper and lower bounded by  $\Delta P_{brk \max}$  and  $\Delta P_{brk \min}$ , respectively ( $\Delta P_{brk \max} \geq 0$  and  $\Delta P_{brk \min} \leq 0$ ). The braking power  $P_{brk}(t)$  is also upper and lower bounded by  $P_{brk \max}$  and  $P_{brk \min} = 0$ , respectively. As a result of including the friction brakes, (1) is modified:

$$P_{mot}(t) = P_{wh}(t)/\eta_{wh}(t) + P_{brk}(t). \quad (5)$$

The SHEV battery state-of-charge,  $SoC(t)$ , may be specified by an integrator-type model:

$$SoC(k) = SoC(k-1) - cP_{bat}(k), \quad (6)$$

where  $SoC(k)$  is a fractional measure of the amount of battery charge at time-step  $k$ ,  $0 \leq SoC(k) \leq 1$ , and  $c$  [kW<sup>-1</sup>] is a battery charge/discharge rate constant. A typical SHEV controller requirement is to have  $SoC(k)$  track a desirable state-of-charge  $SoC_{des}$ , and to ensure that  $SoC(k)$  stays within operational limits  $SoC_{des \max}$  and  $SoC_{des \min}$ .

At each discretized time-step  $k$ , the actual wheel power is:

$$P_{wh \ act}(k) = ((P_{bat}(k) + P_{gen}(k))\eta_{mot}(k) - P_{brk}(k))\eta_{wh}(k), \quad (7)$$

which may not equal driver wheel power demand  $P_{wh}(k)$  due to saturation limits applied after computing (2) and (3). Thus, secondary controller requirements stipulate that actual wheel power quickly recovers to track driver demand, and that the friction brake power matches a desirable value  $P_{brk \ des} = 0$ .

## III. SEGS HIGHLIGHTS

This section recaps the SEGS algorithm, and summarizes how the technique constitutes a Markov chain Monte Carlo method [25] that results in equilibrium higher probabilities for fitter states in a search space (18). The description of the SEGS scheme here is a condensed version of the theoretical results in [22]. Briefly, at every time step, a decision between a current state and a candidate state (which has been generated from the current state using a probability distribution over a set of all states) is made. This decision depends on the comparative fitness of the two states, and is tunable with the help of a parameter that is at 0 when fitness is irrelevant and a random choice suffices, and approaches  $\infty$  when the most fit state is always chosen. The goal is to construct a Markov chain

decision process that has desirable optimization properties in the long run, and that also has desirable responsiveness properties in the short run so that the decision-making process is quick to respond to environmental fluctuations. It turns out that this process can be optimally search-efficient too.

#### A. Problem Definition

Let  $X$  be a search space with elements  $x_i$ ,  $1 \leq i \leq n$ . The search problem seeks a probability mass function  $\phi_X : X \rightarrow \mathbb{R}^+$  that accomplishes the specified objective below, and dynamic transition laws that cause  $X$  to have probability distribution  $\phi_X$ . Let  $z : X \rightarrow Z$  be an unknown, computable, and possibly changing function that we are interested in. The set  $Z$  is a metric space. Suppose that we are given a desired element  $z_{des}$  in the image of  $z$ , and we wish to find  $x \in X$  such that  $\|z(x) - z_{des}\|$  is small (i.e.,  $z(x) \approx z_{des}$ ). Formally, we want a  $\phi_X$  that helps achieve a known expected value  $Y \geq 0$ , i.e.,

$$E_{\phi_X}[\|z(x) - z_{des}\|] = Y. \quad (8)$$

In the above,  $Y$  is effectively a tolerance, i.e., it is the acceptable mean distance between candidates in the image of  $z$  compared to the desired image value. The scheme to find  $\phi_X$  should be efficient in that it trades off prior information about  $X$  for search effort savings as quickly as possible.

Let  $f : Z \rightarrow \mathbb{R}^+$ . We allow the method to employ a function  $F : X \rightarrow \mathbb{R}^+ : x \mapsto F(x) = (f \circ z)(x) = f(z(x))$ , a real-valued, positive fitness function. We desire  $\phi_X(x_i)$  to be responsive to perturbations, i.e., for all  $x_j \in X$ ,

$$\frac{\partial \phi_X(x_i)}{\partial F(x_j)} \neq 0. \quad (9)$$

#### B. The Selective Evolutionary Generation Systems Algorithm

To search in  $X$ , a Markov chain Monte Carlo method is postulated that makes use of a *selective evolutionary generation system*, which is a quintuple  $\Gamma = (X, R, P, G, F)$ , where

- $X$  is the set in which to search,  $X = \{x_1, x_2, \dots, x_n\}$ ;
- $R$  is a “resource” set whose elements can be utilized to transition between elements of  $X$ ,  $R = \{r_1, r_2, \dots, r_m\}$ ;
- $P : R \rightarrow (0, 1]$  is a probability mass function on  $R$ , given by  $P(r_i) = \Pr[\mathcal{R} = r_i] = p_i$ ,  $\sum_{k=1}^m p_k = 1$ ;
- $G : X \times R \rightarrow X$  is a mapping, called a “generation function,” from one element of  $X$  to another using a resource from  $R$ ;
- $F : X \rightarrow \mathbb{R}^+$  is a positive function that evaluates the fitness of elements of  $X$ ;
- $X$  is reachable through  $G$  and  $R$ ; and
- the dynamics of the system are given by

$$\mathcal{X}(t+1) = \text{Select}(\mathcal{X}(t), G(\mathcal{X}(t), \mathcal{R}(t)), N), \quad (10)$$

where  $\text{Select} : X \times X \times \mathbb{N} \rightarrow X$  is a random function such that if  $x_1 \in X$  and  $x_2 \in X$  are any two elements, and  $N \in \mathbb{N}$  is the *level of selectivity*, then

$$\text{Select}(x_1, x_2, N) = \begin{cases} x_1 & \text{with probability } \frac{F(x_1)^N}{F(x_1)^N + F(x_2)^N}, \\ x_2 & \text{with probability } \frac{F(x_2)^N}{F(x_1)^N + F(x_2)^N}. \end{cases} \quad (11)$$

In (10),  $\mathcal{X}(t)$  denotes the realization of a random element from  $X$  at time  $t$ ,  $\mathcal{R}(t)$  denotes the realization of a random resource at time  $t$ ,  $G(\mathcal{X}(t), \mathcal{R}(t))$  denotes the outcome mapped from the realized element from  $X$  at time  $t$  utilizing the resource at time  $t$ , and  $\mathcal{X}(0)$  has a known probability mass function. Also in (10), the probability of realizing an element from  $X$  at some future time given the present realization of an element from  $X$  is conditionally independent of the past time history of  $X$  element realizations. Thus, the dynamics of a selective evolutionary generation system form a discrete-time homogeneous Markov chain.

The *Select* function has a number of interesting properties [22], including that for all  $N$ ,

$$\frac{\Pr[\text{Select}(x_1, x_2, N) = x_1]}{\Pr[\text{Select}(x_1, x_2, N) = x_2]} = \left( \frac{F(x_1)}{F(x_2)} \right)^N. \quad (12)$$

That is, the ratio of the probabilities of selecting any two elements from  $X$  is equal to the ratio of their respective fitnesses raised to the power  $N$ . This property is called “local rationality,” where “rational” refers to the ratio of the probabilities. For any  $x_i, x_j \in X$  and  $r_k \in R$  of the selective evolutionary generation system  $\Gamma = (X, R, P, G, F)$ , we can define the descendency tensor,  $\delta$ , with elements

$$\delta_{ijk} = \begin{cases} 1 & \text{if } x_j = G(x_i, r_k), \quad 1 \leq i \leq n, \quad 1 \leq j \leq n, \\ & \quad 1 \leq k \leq m, \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Hence, the descendency tensor indicates whether it is possible to produce offspring  $x_j$  in one step from progenitor  $x_i$  via generation function  $G$  that employs a resource  $r_k$ . We can use this tensor to create a matrix that represents the conditional probability of transitioning to  $x_j$  from  $x_i$ , by utilizing the probability of selecting each available element in  $R$  and summing over all  $m$  elements. The matrix  $\gamma$ , called the unselective matrix of transition probabilities, has elements

$$\gamma_{ij} = \Pr[\text{offspring is } x_j \mid \text{progenitor is } x_i] \quad (14)$$

$$= \sum_{k=1}^m \delta_{ijk} p_k, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n, \quad (15)$$

and is a stochastic matrix [22]. The matrix of transition probabilities,  $\mathbf{P}$ , has elements

$$P_{ij} = \Pr[\mathcal{X}(t+1) = x_j \mid \mathcal{X}(t) = x_i], \quad (16)$$

$$= \begin{cases} \frac{1}{1 + \left( \frac{F(x_j)}{F(x_i)} \right)^N} \gamma_{ij}, & \forall j \neq i, \\ \gamma_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n \frac{1}{1 + \left( \frac{F(x_j)}{F(x_i)} \right)^N} \gamma_{ij}, & \text{if } j = i, \end{cases} \quad (17)$$

and is also a stochastic matrix [22].

The central idea behind the Selective Evolutionary Generation Systems (SEGS) algorithm is to deploy an ergodic selective evolutionary generation system  $\Gamma = (X, R, P, G, F)$  with symmetric  $\gamma$  (i.e., equiprobable forward and reverse transitions between any pair of elements from  $X$  prior to the selection process) so that the Markov chain that represents the

resultant stochastic dynamics has a row vector of stationary probabilities,  $\pi = [\pi_1 \ \pi_2 \ \dots \ \pi_n]$ ,

$$\pi_i = \frac{F(x_i)^N}{\sum_{k=1}^n F(x_k)^N}, \quad 1 \leq i \leq n. \quad (18)$$

This stationary distribution represents a more general, probabilistic version of the optimization of an objective function. The Markov chain selects the state of maximum fitness with the highest stationary probability, and, in the limit as  $N$  approaches  $\infty$ , this probability is 1. That is,  $N$  tunes the concentration of the stationary probability distribution around the state of maximum fitness, and in the limit as  $N$  approaches  $\infty$ , the problem and solution then revert to one of standard, off-line optimization, i.e., a delta function at an element of  $X$ . In [22], it is also shown that the Markov chain is time-reversible, that the SEGS algorithm is correct, and that increasing  $N$  reduces the mean hitting time to the fittest element.

The Markov chain that represents the stochastic dynamics of a selective evolutionary generation system belongs (as proved in [22]) to a class of time-homogeneous, irreducible, ergodic Markov chains that are said to “behave rationally” with respect to fitness  $F$  with level  $N$ . Such chains are characterized by stationary probability row vectors with elements that satisfy

$$\frac{\pi_i}{\pi_j} = \left( \frac{F(x_i)}{F(x_j)} \right)^N, \quad 1 \leq i \leq n, \quad 1 \leq j \leq n, \quad (19)$$

which is a definition of “global rationality,” where “rational” again refers to the ratio of the probabilities and “global” refers to the stationarity of these probabilities.

Reference [23] discusses how Markov chain rational behavior minimizes a cross-entropy function to yield search entropy. Thereafter, the maximization of this search entropy is investigated, based on results about efficient search from [24] and [26] that specify entropy maximization to eliminate search biases. Such search biases can be induced by, for example, a model that predisposes the optimization process, which causes inefficient search when the model itself is incorrect as a result of internal or external change.

During model-independent search-based optimization with time-varying objective function or time-varying state fitnesses, an exponential fitness function is proved to relate Markov chain rational behavior, search entropy and optimally efficient search [23]. The implication is that a fitness function

$$F(x_i) = e^{-(z(x_i) - z_{des})^2} \quad (20)$$

together with a scheme that makes use of Markov chain rational behavior (for instance, the SEGS technique) guarantees efficient search-based optimization.

Responsiveness of Markov chains that behave rationally is taken as the sensitivity of the stationary distribution to changes in fitness. Specifically, for any time-homogeneous, irreducible, ergodic Markov chain, the extrinsic resilience of state  $x_i$  to changes in the fitness of state  $x_j$ ,  $j \neq i$ , is  $\rho_{ij} = \frac{\partial \pi_i}{\partial F(x_j)}$ , and the intrinsic resilience of state  $x_i$  to changes in its own

fitness is  $\rho_{ii} = \frac{\partial \pi_i}{\partial F(x_i)}$ . The Markov chain is “responsive” if  $\rho_{ij} \neq 0$  for all  $i$  and  $j$ . Reference [22] proves that the level of selectivity has the following asymptotic effect:  $\rho_{ij} \Big|_{N=0} =$

$\rho_{ii} \Big|_{N=0} = 0$ , and  $\lim_{N \rightarrow \infty} \rho_{ij} = \lim_{N \rightarrow \infty} \rho_{ii} = 0$ . That is, standard, off-

line optimization ( $N \rightarrow \infty$ ) and purely random optimization ( $N = 0$ ) are unresponsive. Because the expected hitting time of the element from  $X$  that optimizes fitness also decreases with an increasing level of selectivity  $N$ , a trade-off exists between this expected hitting time and responsiveness, with the trade-off controlled by  $N$ .

Reference [22] goes on to prove that Markov chain rational behavior is a sufficient condition for responsiveness, while ergodicity is a necessary condition for responsiveness. In addition, [22] also provides four equations to analyze the effect of changes in fitness on elements of the matrix of transition probabilities,  $\mathbf{P}$ . These equations demonstrate that, unlike gradient ascent optimization where the transition to another element from  $X$  is directly proportional to the fitness value, optimization with Markov chain rational behavior is reminiscent of the retardation property in the original rational behavior [27]; the stochastic process “slows down” transitions in more favorable fitness conditions to take advantage of the external environment.

#### IV. SHEV CONTROL WITH SEGS

We consider the SHEV model with friction brakes from Section II with the following parameters:  $\eta_{wh} = 0.95$ ;  $\Delta P_{gen \ max} = 11$  kW;  $\Delta P_{gen \ min} = -11$  kW;  $P_{gen \ max} = 80$  kW;  $P_{gen \ min} = -5$  kW;  $\eta_{mot} = 0.95$ ;  $P_{bat \ max} = 40$  kW;  $P_{bat \ min} = -30$  kW;  $c = 1.25 \times 10^{-5}$  kW<sup>-1</sup>;  $\Delta P_{brk \ max} = 24$  kW;  $\Delta P_{brk \ min} = -24$  kW;  $P_{brk \ max} = 24$  kW;  $P_{brk \ min} = 0$  kW;  $P_{brk \ des} = 0$  kW;  $SoC_{des} = 0.5$ ;  $SoC_{des \ max} = 0.6$ ; and  $SoC_{des \ min} = 0.4$ . We also specify the following selective evolutionary generation system for use with the SEGS algorithm. The set  $X$  is the couple of  $P_{gen}(k)$  [kW] and  $P_{brk}(k)$  [kW] with respective elements from the sets:  $\{P_{gen \ min}, P_{gen \ min} + 1, \dots, P_{gen \ max} - 1, P_{gen \ max}\}$  and  $\{P_{brk \ min}, P_{brk \ min} + 2, \dots, P_{brk \ max} - 2, P_{brk \ max}\}$ . The set  $R$  is the set product of feasible variations of  $\Delta P_{gen}(k)$  [kW]:  $\{-22, -20, \dots, -4, -2, -1, 0, 1, 2, 4, \dots, 20, 22\}$  and of  $\Delta P_{brk}(k)$  [kW]:  $\{-24, -22, \dots, -2, 0, 2, \dots, 22, 24\}$ . Thus, the generation function  $G$  is (2) and (4) as long as  $P_{gen \ min} \leq P_{gen}(k) \leq P_{gen \ max}$  and  $P_{brk \ min} \leq P_{brk}(k) \leq P_{brk \ max}$ ; otherwise,  $P_{gen}(k) = P_{gen}(k-1)$  if the saturation on (2) is active and  $P_{brk}(k) = P_{brk}(k-1)$  if the saturation on (4) is active.

The above is a modified version of a random walk over a discretized search space, where the modification involves selection dynamics described by the *Select* function to produce a selective evolutionary generation system. Since the primary objective is for  $SoC(t)$  to track  $SoC_{des}(t)$  using efficient search, we choose to use a fitness function

$$F(\Delta P_{gen}(k)) = \exp\left(-\left(K_f(SoC_{des}(k) - SoC(k))\right)^2\right), \quad (21)$$

where  $K_f = 15$  and  $SoC(k)$  is the output of the SHEV battery model in simulation. Because  $SoC(k)$  is determined

external to the SEGS algorithm, it can be a sensor reading in practice. The fitness function is chosen such that the condition  $SoC(k) = SoC_{des}(k)$  has maximal unit fitness, and the conditions  $SoC(k) = SoC_{des\ max}(k)$  and  $SoC(k) = SoC_{des\ min}(k)$  have only 10% of maximum fitness. This low fitness ensures “high-gain control” at or beyond  $SoC$  constraint boundaries. The initial level of selectivity is  $N = 100$ .

Sample results of SHEV control with the SEGS technique for  $SoC(0) = 0.47$  and  $P_{brk}(0) = 0$  are presented in Figs. 1–3 for a step-like power-demand profile, a portion of the UDDS city power-demand profile, and a portion of the US06 highway power-demand profile, respectively. In Fig. 1, it is clear that the desired battery state-of-charge is tracked, although the chosen level of selectivity permits excursions of the state-of-charge to undesirable values. During intervals of accurate state-of-charge tracking, the generator actuation is pulse-width-modulated. There are also instances when the actual wheel power does not match the demanded power, and when the friction brakes are needlessly activated. This controller behavior may be attributed to a fitness function that does not enforce wheel and brake power requirements.

Low wheel power requests are made of the controller by the UDDS city power-demand profile in Fig. 2. The state-of-charge oscillates between 0.46 and 0.53 because of alternate use of the generator and battery to satisfy the low power requests. The available wheel power always meets or exceeds the demand. Compared to the related plot in [17] where the SHEV is operated with high efficiency, the state-of-charge values there decrease from 0.47 to 0.43 over the same period.

In Fig. 3, the SEGS controller satisfies the high wheel power requests typical of the US06 highway power-demand profile with the generator while maintaining the state-of-charge between 0.42 and 0.49. Here, an occasional lack of available wheel power is more pronounced because the wheel power demand is not enforced. Similarly, there is an interval when brake power is zero despite excess available wheel power. Compared to the related plot in [17] where the SHEV is operated with high efficiency, a lower variation exists here over the same period (plotted there from 110s to 200s); the lowest state-of-charge there is 0.44 and the highest is 0.53.

Fig. 4 corroborates the hypothesis by demonstrating improved state-of-charge reference tracking on the step-like power demand profile at higher  $N$ ; here,  $N = 1000$ . The pulse-width modulation of the generator is also more apparent, with this behavior slightly altered by friction brake action.

To rectify inadequate available wheel power and inappropriate friction braking, we empirically demonstrate a modified efficient-search SEGS algorithm that enforces multiple criteria through a weighted sum of exponential fitness functions. For instance, consider the fitness function,  $F = 0.5 \exp\left(-\left(K_f (SoC_{des}(k) - SoC(k))\right)^2\right) + 0.5 \exp\left(-\left(K_f \left(\frac{P_{wh}(k) - P_{wh\ acr}(k)}{P_{gen\ max} + P_{bat\ max}}\right)\right)^2\right)$ , where the second term ensures that the wheel power demand is met. This term includes a normalization factor consisting of total generator and battery power. Sample SHEV control performance ( $N = 100$ ) of satisfactory state-of-charge tracking and an accurate

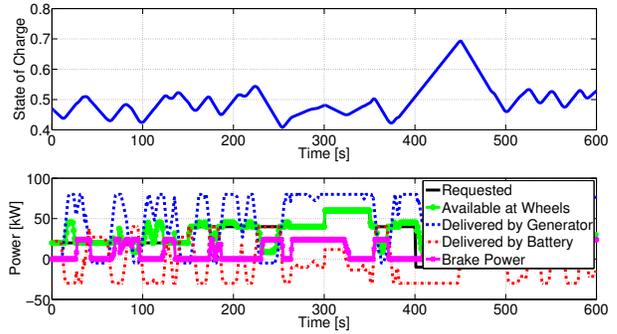


Fig. 1. Sample SEGS controller performance on a step-like power-demand profile.

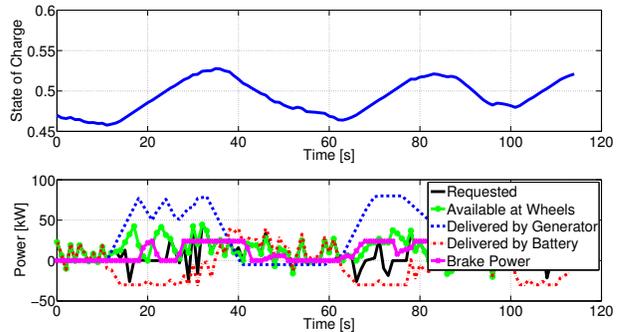


Fig. 2. Sample SEGS controller performance on a portion of the UDDS city power-demand profile.

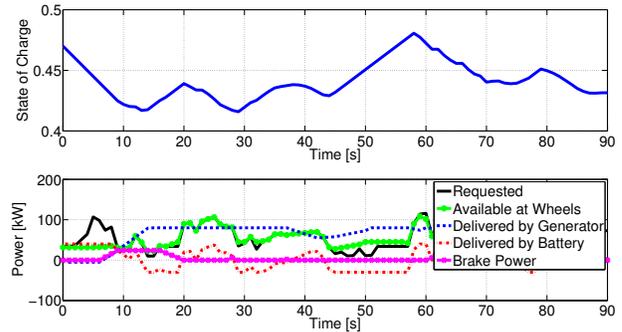


Fig. 3. Sample SEGS controller performance on a portion of the US06 highway power-demand profile.

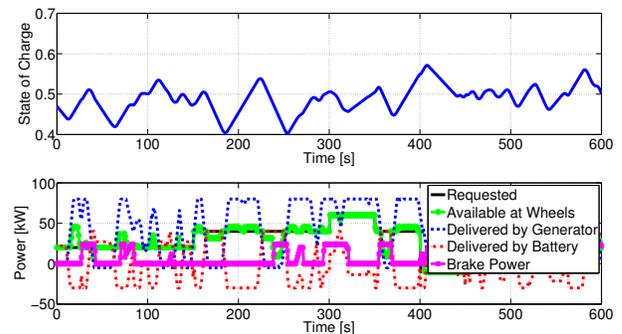


Fig. 4. Sample SEGS controller performance on a step-like power-demand profile,  $N = 1000$ .

realization of wheel power demand is depicted in Fig. 5.

When the state-of-charge weight is reduced from 0.5 to 0.4 and a third term for friction brakes such

as  $0.1 \exp\left(-\left(1.5(P_{brk\ des}(k) - P_{brk}(k)/P_{brk\ max})\right)^2\right)$  is included (the 1.5 factor yields 10% fitness at maximum braking), control trajectories like that of Fig. 6 result. Here, the desired state-of-charge, driver wheel power demand, and infrequent friction brake use criterion are all met.

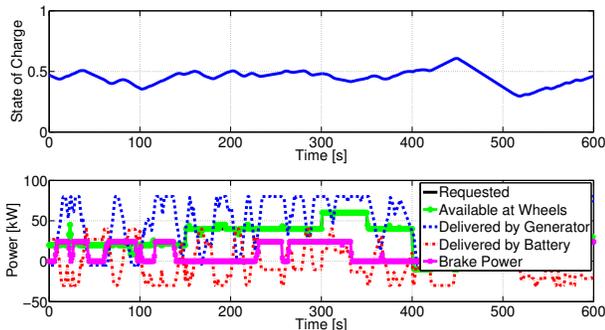


Fig. 5. Sample SEGS controller performance on a step-like power-demand profile with a fitness function that enforces both wheel power demand and desirable state-of-charge.

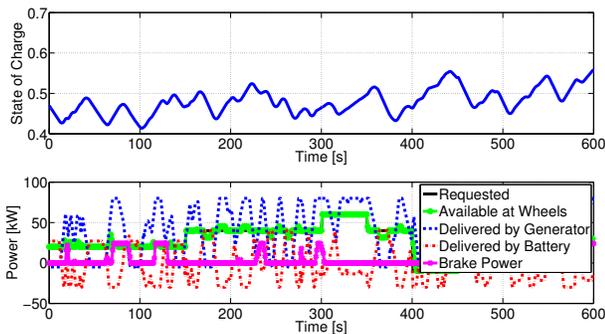


Fig. 6. Sample SEGS controller performance on a step-like power-demand profile with a fitness function that enforces desirable state-of-charge, wheel power demand, and reduced friction brake use.

## V. CONCLUDING REMARKS

This paper has illustrated the efficacy of computationally-inexpensive selective evolutionary generation in managing SHEV energy and power on-line. The technique is pursued because it constitutes a model-independent control policy that yields a target distribution of high probabilities for fit SHEV states and low probabilities for unfit states.

Future work includes exploring dynamically changing levels of selectivity as well as responsiveness to driving history. Overall powertrain efficiency improvement will also be demonstrated, as will increased fuel economy.

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